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TECHNICAL NOTES

Momentum and heat transport on a continuous flat surface moving in a parallel stream

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1. INTRODUCTION

IN RECENT years the boundary layer flow on a continuous flat surface, where both the free stream and plate velocities are constant and are moving in the same direction, has been studied by Abdelhafez [1] and Chappidi and Gunnerson [2]. Their analyses consider two cases separately, where the velocity of the moving wall U_w is greater or less than the free stream velocity U_∞. Accordingly for the two cases of $U_\infty < U_w$ and $U_w < U_\infty$ two sets of boundary value problems were formulated and a numerical solution [1] and integral solution [2] were reported.

This note describes a method to formulate a single set of equations employing a composite velocity [3], irrespective of whether $U_w > U_\infty$ or $U_w < U_\infty$.

2. ANALYSIS

The flow field, shown in Fig. 1, is steady in a Cartesian coordinate system (x, y) fixed in space. Both the free stream velocity U_∞ and plate velocity U_w are constants and are moving in the same direction. Further, the plate is maintained at a constant temperature T_w. The temperature of the ambient fluid is T_∞ and the Prandtl number of fluid is σ. The momentum and thermal boundary layer equations are

$$u_x + v_y = 0 \tag{1}$$

$$uu_x + vu_y = \nu u_{yy} \tag{2}$$

$$uT_x + vT_y = \sigma^{-1}\nu T_{yy} \tag{3}$$

subjected to the boundary conditions

$$y = 0 \quad u = U_w \quad v = 0, \quad T = T_w \tag{4a}$$

$$y \to \infty \quad u \to U_\infty \quad T \to T_\infty. \tag{4b}$$

Based on the similarity variables

$$\psi = (2\nu x U)^{1/2} f(\eta), \quad \eta = y(U/2\nu x)^{1/2}$$
$$T = T_\infty + (T_w - T_\infty) g(\eta) \tag{5}$$

where U is some suitable reference velocity (to be specified later), the momentum boundary layer equations reduce to

$$f''' + ff'' = 0 \tag{6}$$

$$f(0) = 0, \quad f'(0) = U_w/U, \quad f'(\infty) = U_\infty/U. \tag{7}$$

Following Abdelhafez [1] where the largest of the two velocities was adopted as the reference velocity, i.e.

$$U = \begin{cases} U_w & \text{if} \quad U_w > U_\infty \\ U_\infty & \text{if} \quad U_\infty > U_w \end{cases} \tag{8}$$

the boundary conditions (7) lead to two sets of boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = U_\infty/U_w, \quad U_w > U_\infty \tag{9}$$

$$f(0) = 0, \quad f'(0) = U_w/U_\infty, \quad f'(\infty) = 1, \quad U_\infty > U_w. \tag{10}$$

The solutions of (6) subject separately to (9) and (10) have been reported in refs. [1, 2]. Earlier the mathematical problem (6) subject to (10) arose in Mirels work on the boundary layer behind a moving shock wave [4], Clauser [5] on the equilibrium of the outer layer of a turbulent boundary layer, Kalinin *et al.* [6] in film boiling or condensation on a flat surface and Merkin [7] in mixed convection on a flat surface embedded in a Darcian porous medium.

In the present work, the reference velocity U is taken as a composite velocity [3]

$$U = U_\infty + U_w. \tag{11}$$

Based on (11) and similarity variables (5) the momentum boundary layer equations reduce to

$$f''' + ff'' = 0 \tag{12}$$

$$f(0) = 0, \quad f'(0) = 1 - \varepsilon, \quad f'(\infty) = \varepsilon \tag{13}$$

where ε is a parameter given by

$$\varepsilon = U_\infty/(U_\infty + U_w), \quad U_w + U_\infty \neq 0. \tag{14}$$

For classical Blasius problem $\varepsilon = 1$ and for the Sakiadis [8] problem $\varepsilon = 0$. The case where both the free stream and the plate velocities are in the same direction correspond to $0 \leqslant \varepsilon \leqslant 1$. If $\varepsilon > 1$, the free stream is directed towards the positive x-direction while the plate moves towards the negative x-direction. If $\varepsilon < 0$, the free stream is directed towards the negative x-direction while the plate moves towards the positive x-direction.

The thermal boundary layer equations based on (5) and (11) lead to

$$g'' + \sigma f g' = 0 \tag{15}$$

$$g(0) = 1, \quad g(\infty) = 0. \tag{16}$$

The wall shear stress τ_w and heat transfer q_w are given by

$$\tau_w = \mu \left[\frac{(U_\infty + U_w)^3}{2\nu x} \right]^{1/2} f''(0)$$

$$q_w = -\mu\sigma^{-1} \left(\frac{U_\infty + U_w}{2\nu x} \right)^{1/2} g'(0). \tag{17}$$

The non-existence of solution for $\varepsilon > \varepsilon_0$ is proved in the Appendix. The numerical solutions show that for $\varepsilon < 1$ the solutions are unique whereas for $1 \leqslant \varepsilon < \varepsilon_0$ the solutions are dual.

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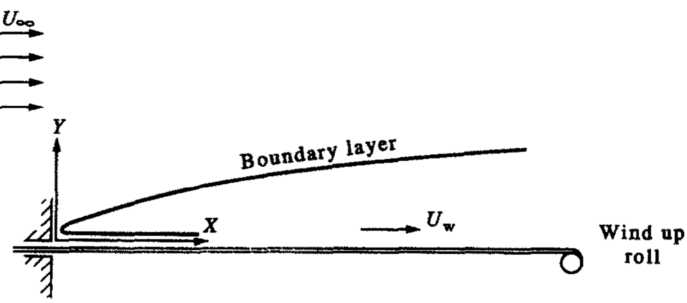

FIG. 1. Flow model of a moving sheet subjected to parallel free stream.

3. RESULTS AND DISCUSSION

Numerical solutions to the momentum boundary layer equation (12) subject to boundary conditions (13) have been obtained. For $0 \leqslant \varepsilon \leqslant 1$ the non-dimensional velocity distribution $(u-U_\infty)/(U_w-U_\infty)$ is displayed in Fig. 2. The results compare well for $\varepsilon = 0$ with Sakiadis solution and $\varepsilon = 1$ with the Blasius solution. The non-dimensional velocity profiles become steeper near the wall as ε increases. The characteristic numerical solution for $f''(0)$ and $b = \lim_{\eta\to\infty}(\varepsilon\eta - f)$ are displayed in Fig. 3. For $\varepsilon < 1$, the behaviour of $f''(0)$ is monotonic with ε; it is negative for $\varepsilon < 0.5$, positive for $\varepsilon > 0.5$ and zero for $\varepsilon = 0.5$. The change in sign of $f''(0)$ at $\varepsilon = 0.5$ is associated with a corresponding change of sign for normal component of velocity; which is directed towards the wall for $0 \leqslant \varepsilon < 0.5$ and away from the wall for $0.5 < \varepsilon \leqslant 1$. Physically, the positive sign of $f''(0)$ for $\varepsilon > 0.5$ means that the fluid exerts a dragging force on the sheet and negative sign for $\varepsilon < 0.5$ implies that the fluid is being dragged by the sheet. Here zero skin friction does not mean separation and corresponds to the parallel flow $U_\infty = U_w$. Further, Fig. 3, shows that there exists a critical value $\varepsilon_0 = 1.548$ that is marked by an arrow on the ε-axis. The solution is unique for $\varepsilon < 1$, dual for $1 \leqslant \varepsilon < \varepsilon_0$ and no solution for $\varepsilon > \varepsilon_0$. As $\varepsilon \to 1^+$ the dual solution corresponds to $f''(0) \to 0$ and $b \to \infty$ rapidly.

The approximate solution of momentum boundary layer, described in the Appendix compares well with the numerical solution for $\varepsilon \lessgtr 1$. This solution, however, is not workable for $1 < \varepsilon < \varepsilon_0$ (because the assumed velocity profile is not appropriate) nevertheless it shows that there is no solution for $\varepsilon > \varepsilon_0$. Consequently there is an upper bound on $\varepsilon \leqslant \varepsilon_0$ for which solutions exist.

For $\varepsilon < 0$, $U_\infty < 0$ the free stream velocity is directed towards the negative x-direction, the numerical solutions of the momentum boundary layer are also displayed in Fig. 3 by dotted lines. It was found that the numerical solutions equations (12) and (13) do not decay exponentially at large η. Therefore for $\varepsilon < 0$ the integral solution of the Appendix can be usefully employed.

For $\varepsilon = 1/2$ the closed form solutions of momentum and energy equations are

$$f = \eta/2, \quad f''(0) = 0$$

$$g = \mathrm{erfc}(\eta\sigma^{1/2}/2), \quad g'(0) = -(\sigma/\pi)^{1/2}. \tag{18}$$

For $\sigma = 1$ the solution of energy equation is related to momentum equation by

$$g(\eta) = [f'(\eta) - \varepsilon]/(1-2\varepsilon) \tag{19}$$

and this leads to the Reynolds analogy between momentum and heat transfer. Numerical solutions to the thermal boundary layer equation have been obtained for various values of Prandtl number σ ranging from 0.1 to 100. The results for temperature profiles for $\sigma = 0.72$ and various values of ε are displayed in Fig. 4. The temperature gradient $g'(0)$ for $\sigma = 0.72$ and various values of ε are displayed in Fig. 5. Their behaviour is qualitatively similar to $f''(0)$ and no additional

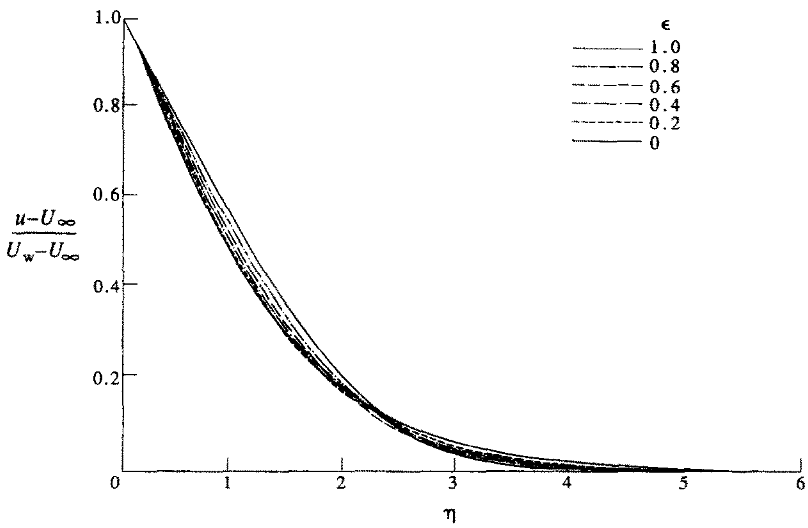

FIG. 2. Non-dimensional velocity profile on a moving sheet subjected to a parallel uniform free stream in the same direction ($0 \leqslant \varepsilon \leqslant 1$).

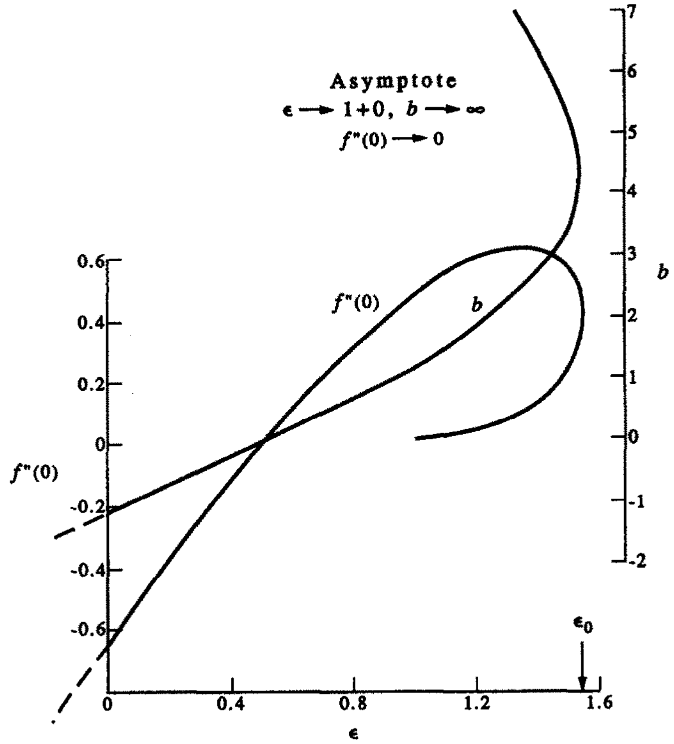

FIG. 3. Velocity gradient at the sheet $f''(0)$ and displacement thickness $b = \lim_{\eta\to\infty} (\varepsilon\eta - f)$ as a function of composite velocity parameter ε. The critical value $\varepsilon_0 = 1.548$ is marked by an arrow on ε-axis beyond which no solution exists. $\varepsilon > 1$, the sheet is moving in the negative x-direction ($U_w < 0$); $\varepsilon < 0$, the free stream is in negative x-direction ($U_\infty < 0$); for $0 \leqslant \varepsilon \leqslant 1$ both free stream and wall velocities are in same direction.

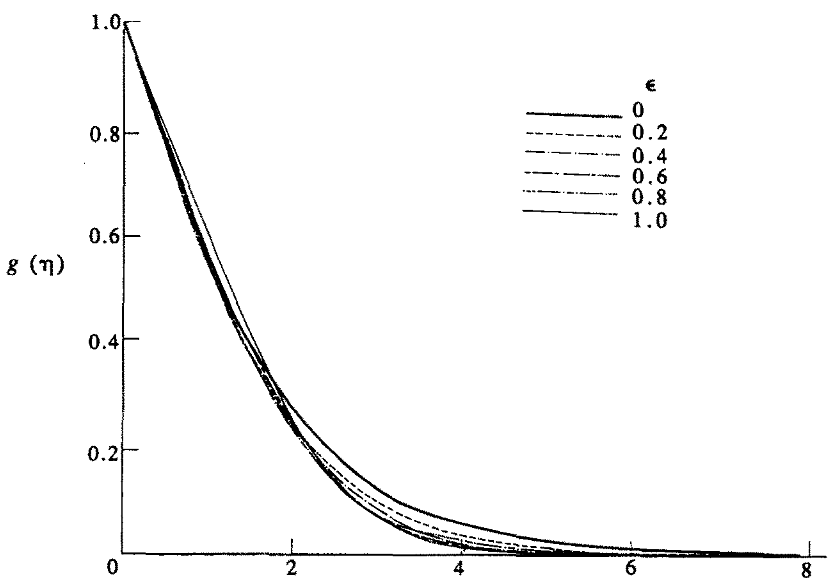

FIG. 4. Temperature profile for $\sigma = 0.72$ on a moving sheet subjected to a free stream in the same direction ($0 \leqslant \varepsilon \leqslant 1$).

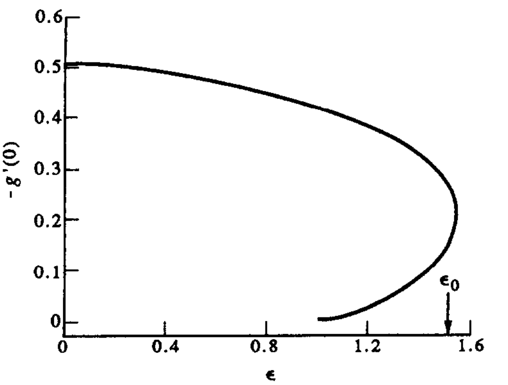

FIG. 5. Temperature gradient $g'(0)$ for various values of ($\sigma = 0.72$). The critical value $\varepsilon_0 = 1.548$ is marked by an arrow on the ε-axis beyond which no solution exists.

comment is needed. The effects of σ on the temperature gradient at the wall for $0.1 \leqslant \sigma \leqslant 100$ is displayed in Fig. 6. For fixed ε, the magnitude of $g'(0)$ increases with Prandtl number σ. On the other hand, when σ is held fixed then for every increase of ε, the magnitude of $g'(0)$ decreases for large σ, and $g'(0)$ increases for small σ. In order to further explore this behaviour the asymptotic analysis at limiting Prandtl numbers was carried out and the results are summarized below. The heat transfer rate for $\sigma \to 0$ is

$$g'(0) = -(2\sigma\varepsilon/\pi)^{1/2} + 2b\sigma/\pi + \cdots \quad \varepsilon \neq 0 \quad (20a)$$

$$= b\sigma + \cdots \quad \varepsilon = 0. \quad (20b)$$

The results for $\sigma \to 0$ show that $g'(0)$ is of $0(\sigma^{1/2})$ for $\varepsilon \neq 0$ and of $0(\sigma)$ for $\varepsilon = 0$. This implies that at low σ the heat transfer rate $g'(0)$ can be enhanced by imposing a free stream. This is because at low σ the thermal boundary layer thickness is very much larger than the momentum boundary layer. To the lowest order, the thermal boundary layer depends on the flow outside the boundary layer and momentum boundary layer can be neglected. Further, when ε is of $0(\sigma)$ the two terms in (20a) are of same order. At a particular point $\varepsilon = k\sigma$, $k = [(\pi-2)b]^2/2\pi$, the two results (20a) and (20b) become the same.

The heat transfer rate for $\sigma \to \infty$ is

$$g'(0) = -[2\sigma(1-\varepsilon)/\pi]^{1/2} + 0(\sigma^{1/2}), \quad \varepsilon \neq 1 \quad (21a)$$

$$= -\frac{3}{\Gamma(\frac{1}{3})}\left(\frac{\sigma}{6}f''(0)\right)^{1/3} + 0(\sigma^{1/3}), \quad \varepsilon = 1. \quad (21b)$$

The results show that $g'(0)$ is of $0(\sigma^{1/2})$ for $\varepsilon \neq 1$ and of $0(\sigma^{1/3})$ for $\varepsilon = 1$. This means that at high σ the heat transfer rate $g'(0)$ is reduced when free stream is imposed on the moving sheet. This is because at high σ the thermal boundary layer is very thin and to the lowest order it is governed by the flow in the immediate vicinity of the sheet and the imposition of the free stream reduces the velocity gradient at the wall.

Thus, the results of Fig. 6 are in confirmity with the asymptotic behaviour for $\sigma \to 0$ and $\sigma \to \infty$. From Fig. 6 one can say that, there exists a $\sigma_0(\varepsilon)$ such that for $\sigma > \sigma_0$, $g'(0)$ decreases with ε and $\sigma < \sigma_0$, $g'(0)$ increases with ε. Further, $g'(0)$ remains practically constant at and around $\sigma = \sigma_0$. A crude estimate of σ_0 can be taken as 0.5.

4. CONCLUSIONS

1. The boundary layer on a moving sheet subjected to a parallel free stream can be studied by single set of equations (12) and (13), which are in contrast to earlier works [1, 2].

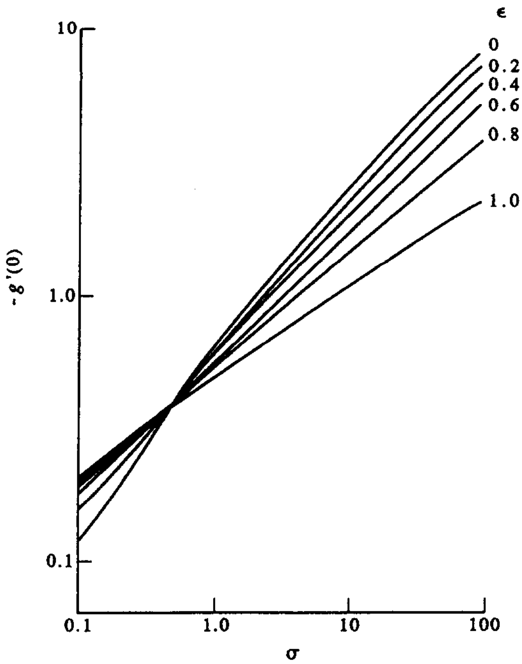

FIG. 6. Temperature gradient at the sheet $g'(0)$ in the thermal boundary layer for various values of Prandtl number $0.1 \leqslant \sigma \leqslant 100$ and $\varepsilon = 0(0.2)1$.

2. The velocity gradient at wall $f''(0)$ is positive (fluid exerts a drag force on the sheet) for $\varepsilon > 0.5$, and negative (fluid is being dragged by the moving sheet) for $\varepsilon < 0.5$. Further, $f''(0) = 0$ at $\varepsilon = 0.5$ does not imply separation but the fact that the solution corresponds to the parallel flow and the momentum boundary layer is absent.

3. The solution is unique for $\varepsilon < 1$ and dual for $1 \leqslant \varepsilon < \varepsilon_0$. There is no solution for $\varepsilon > \varepsilon_0$ implying that the theory is more complicated than the one considered here.

4. The temperature gradient at the moving wall $g'(0)$ increases at low σ and decreases at high σ when free stream is imposed on the moving sheet. Further, $g'(0)$ remains practically independent of ε (free stream) around $\sigma = 0.5$ for $0 < \varepsilon < 1$.

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APPENDIX: INTEGRAL ANALYSIS

An integral of momentum equation (12) between limits zero to ∞ gives

$$f''(0) = \int_0^\infty \varepsilon f' - f'^2 \, \mathrm{d}\eta. \tag{A1}$$

Introducing a function ϕ such that

$$\phi(\zeta) = \frac{u-U_w}{U_\infty - U_w} = \frac{f'+\varepsilon-1}{2\varepsilon-1}, \quad \zeta = \frac{\eta}{A} \tag{A2}$$

$$\phi(0) = 0, \quad \phi(\infty) = 1.$$

The momentum integral (A1) yields

$$A^2 = \phi^1(0)/[B_1 - B_2 - \varepsilon(B_1 - 2B_2)] \tag{A3}$$

where

$$B_1 = \int_0^\infty 1-\phi \, \mathrm{d}\zeta, \quad B_2 = \int_0^\infty \phi - \phi^2 \, \mathrm{d}\zeta.$$

Using the above results it can be shown that

$$f''(0) = (2\varepsilon-1)[\phi^1(0)(B_1 - B_2 - \varepsilon(B_1 - 2B_2))]^{1/2}$$
$$b = (2\varepsilon - 1)B_1 A. \tag{A4}$$

The relation (A4) shows that a solution does not exist for $\varepsilon > \varepsilon_0$ where ε_0 is given by

$$\varepsilon_0 = 1 + (H-2)^{-1} \tag{A5}$$

where $H = B_1/B_2$ is the shape factor ($H > 2$, $\varepsilon_0 > 1$).

If one considers a trial velocity profile

$$\phi(\zeta) = (3\zeta - \zeta^3)/2, \quad \zeta \leqslant 1; \quad \phi(\zeta) = 1, \quad \zeta > 1$$
$$B_1 = 3/8, \quad B_2 = 39/280$$

then $\varepsilon_0 = 2.46$ and the solution is given by

$$f''(0) = (2\varepsilon - 1)(0.3536 - 0.1446\varepsilon)^{1/2} \tag{A6}$$

and for the trial profile

$$\phi(\zeta) = 2\zeta - 2\zeta^3 + \zeta^4, \quad \zeta \leqslant 1; \quad \phi(\zeta) = 1, \quad \zeta > 1$$
$$B_1 = 3/10, \quad B_2 = 37/315$$

$\varepsilon_0 = 2.81$ and the solution is given by

$$f''(0) = (2\varepsilon - 1)(0.365 - 0.130\varepsilon)^{1/2}. \tag{A7}$$

The expression (A7) includes the two results (10) and (18) of ref. [2] as special cases. These solutions are good for $\varepsilon \leqslant 1$ as for $\varepsilon = 1$ the error is about 2% and $\varepsilon = 0$ the error is about 5%. As $\varepsilon \to -\infty$, the asymptotic behaviour of integral solution (A7) leads to

$$f''(0) \to -0.72\,(-\varepsilon)^{3/2}$$
$$b \to -0.83\,(-\varepsilon)^{1/2}. \tag{A8}$$

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Three-dimensional natural convection in a vertical porous layer with hexagonal honeycomb core of negligible thickness

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INTRODUCTION

HONEYCOMB structures are often used in thermal insulating walls. Inside such walls, the main mechanisms of heat transfer are by natural convection and radiation. A number of studies for natural convection heat transfer in such an air layer were investigated by Asako *et al.* [1–3]. If the air layer is filled with thermal insulations, such as glass wool, both convective and radiative heat transfer rates will decrease. A numerical analysis was reported by Asako *et al.* [4] to investigate heat transfer characteristics by natural convection in such a porous layer. The results were obtained for both conductive and adiabatic honeycomb core wall thermal boundary conditions. These conditions exist when the honeycomb core walls are good conductors and thick, and also when they are thermally insulated. For thermal insulating walls, it is required to reduce the heat loss through the honeycomb core walls. Then, the honeycomb core walls should be made as thin as possible to reduce the heat conduction through it. The motivation for the present study is to analyse the case where the honeycomb core walls are assumed to be poor conductors and thin, such that the thermal wall boundary conditions approach the so-called 'no-thickness' wall boundary condition dictated by Nakamura *et al.* [5]. These three boundary conditions, 'conduction', 'adiabatic', and 'no-thickness', can be considered as three idealized thermal boundary conditions. Therefore, the heat transfer rate in a practical porous layer will have a value that lies within these three conditions.

The numerical methodology used in this study utilizes an